

# Helicity Basis for Spin 1/2 and 1, and Discrete Symmetry Operations

Valeri V. Dvoeglazov <sup>†</sup> and J. L. Quintanar González <sup>†</sup>

<sup>†</sup> Universidad de Zacatecas

A. P. 636, Suc. UAZ, C. P. 98062, Zacatecas, Zac., México

E-mail: valeri@cantera.reduaz.mx, el\_leo\_xyz@yahoo.com.mx

**Abstract.** We study the theory of the  $(1/2, 0) \oplus (0, 1/2)$  and  $(1, 0) \oplus (0, 1)$  representations of the Lorentz group in the helicity basis. The helicity eigenstates are *not* the parity eigenstates. This is in accordance with the idea of Berestetskii, Lifshitz and Pitaevskii. The properties of the helicity eigenstates with respect to the charge conjugation and the  $CP$ - conjugation are also considered.

## 1. Introduction.

What are motivations for this work? First of all, Berestetskii, Lifshitz and Pitaevskii stressed [1]: “... the orbital angular momentum  $\mathbf{l}$  and the spin  $\mathbf{s}$  of a moving particle are not separately conserved. Only the total angular momentum  $\mathbf{j} = \mathbf{l} + \mathbf{s}$  is conserved. The component of the spin in any fixed direction (taken as  $z$ -axis) is therefore also not conserved, and cannot be used to enumerate the polarization (spin) states of moving particle.” Moreover, they made conclusion that the helicity eigenstates are *not* the parity eigenstates for any spin [1, p.59], see also [2]. Next, working with the  $\Psi_{(6)} = \text{column}(\mathbf{E} + i\mathbf{B}, \mathbf{E} - i\mathbf{B})$  in the Weinberg-Tucker-Hammer formalism [3, 4, 5] I found that upon rotation of  $\Psi$  we can obtain much more equations for the antisymmetric tensor (AST) field of the 2nd rank than in the accustomed Proca formalism. Some of them imply parity-violating transitions (i. e., contain the dual tensor and the axial-vector 4-potential). Then, we generalized the Dirac formalism [6, 7, 8, 9] and the Bargmann-Wigner formalism [10, 11, 12].

In this paper we are going to study transformations from the standard basis to the helicity basis in the Dirac theory and in the  $(1, 0) \oplus (0, 1)$  Sankaranarayanan-Good theory [13, 14]. The spin basis rotation *changes* the properties of the corresponding states with respect to parity. The parity is a physical quantum number; so, we try to extract corresponding physical contents from considerations of the various spin bases.

## 2. The $(1/2, 0) \oplus (0, 1/2)$ case.

We know that in the  $(1/2, 0) \oplus (0, 1/2)$  representation the helicity operator  $\sigma \cdot \hat{\mathbf{p}}/2 \otimes I_2$ ,  $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$ , commutes with the Hamiltonian (more precisely, the commutator is equal to zero when acting on the one-particle plane-wave solutions). Previously, the 4-spinors have been studied very well when the spin basis has been chosen in such a way that they were eigenstates of the  $\hat{\mathbf{S}}_3$  operator, e. g., ref. [15]. The helicity basis case has not been studied almost at all (see, however, refs. [2, 16, 17]). The 2-eigenspinors of the helicity operator  $\frac{1}{2}\sigma \cdot \hat{\mathbf{p}} = \frac{1}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{+i\phi} & -\cos \theta \end{pmatrix}$  can be defined as follows [18, 19]:

$$\phi_{\frac{1}{2}\uparrow} = \begin{pmatrix} \cos \frac{\theta}{2} e^{-i\phi/2} \\ \sin \frac{\theta}{2} e^{+i\phi/2} \end{pmatrix}, \quad \phi_{\frac{1}{2}\downarrow} = \begin{pmatrix} \sin \frac{\theta}{2} e^{-i\phi/2} \\ -\cos \frac{\theta}{2} e^{+i\phi/2} \end{pmatrix}, \quad (1)$$

for  $\pm 1/2$  eigenvalues, respectively.

We start from the Klein-Gordon equation, generalized for describing the spin-1/2 particles (i. e., two additional degrees of freedom);  $c = \hbar = 1$ , see ref. [20].

$$(E + \sigma \cdot \mathbf{p})(E - \sigma \cdot \mathbf{p})\phi = m^2\phi. \quad (2)$$

It can be re-written in the form of the set of two first-order equations for 2-spinors as in [20]. Simultaneously, we observe that they may be chosen as eigenstates of the helicity operator which present in (2). If the  $\phi$  spinors are defined by the equation (1), then we can construct the corresponding  $u$ - and  $v$ - 4-spinors:

$$\begin{aligned} u_{\uparrow} &= N_{\uparrow}^+ \begin{pmatrix} \phi_{\uparrow} \\ \frac{E-p}{m} \phi_{\uparrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{E+p}{m}} \phi_{\uparrow} \\ \sqrt{\frac{m}{E+p}} \phi_{\uparrow} \end{pmatrix}, \quad u_{\downarrow} = N_{\downarrow}^+ \begin{pmatrix} \phi_{\downarrow} \\ \frac{E+p}{m} \phi_{\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{m}{E+p}} \phi_{\downarrow} \\ \sqrt{\frac{E+p}{m}} \phi_{\downarrow} \end{pmatrix}, \quad (3) \\ v_{\uparrow} &= N_{\uparrow}^- \begin{pmatrix} \phi_{\uparrow} \\ -\frac{E-p}{m} \phi_{\uparrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{E+p}{m}} \phi_{\uparrow} \\ -\sqrt{\frac{m}{E+p}} \phi_{\uparrow} \end{pmatrix}, \quad v_{\downarrow} = N_{\downarrow}^- \begin{pmatrix} \phi_{\downarrow} \\ -\frac{E+p}{m} \phi_{\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{m}{E+p}} \phi_{\downarrow} \\ -\sqrt{\frac{E+p}{m}} \phi_{\downarrow} \end{pmatrix} \end{aligned} \quad (4)$$

where the normalization to the unit ( $\pm 1$ ) was used. They satisfy the Dirac equation with  $\gamma$ 's to be in the spinorial representation. One can prove that the matrix  $P = \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  can also be used in the parity operator as well as in the original Dirac basis [21]. Of course, it is possible to expand the 4-spinors defined in the parity basis in linear superpositions of the helicity basis 4-spinors and to find corresponding coefficients of the expansion:

$$u_{\sigma}(\mathbf{p}) = A_{\sigma\lambda} u_{\lambda}(\mathbf{p}) + B_{\sigma\lambda} v_{\lambda}(\mathbf{p}), \quad (5)$$

$$v_{\sigma}(\mathbf{p}) = C_{\sigma\lambda} u_{\lambda}(\mathbf{p}) + D_{\sigma\lambda} v_{\lambda}(\mathbf{p}). \quad (6)$$

Neither  $A$  nor  $B$  are unitary:

$$A = (a_{++} + a_{+-})(\sigma_{\mu} a^{\mu}) + (-a_{-+} + a_{--})(\sigma_{\mu} a^{\mu}) \sigma_3, \quad (7)$$

$$B = (-a_{++} + a_{+-})(\sigma_{\mu} a^{\mu}) + (a_{-+} + a_{--})(\sigma_{\mu} a^{\mu}) \sigma_3, \quad (8)$$

where

$$a^0 = -i \cos(\theta/2) \sin(\phi/2) \in \Im m, \quad a^1 = \sin(\theta/2) \cos(\phi/2) \in \Re e, \quad (9)$$

$$a^2 = \sin(\theta/2) \sin(\phi/2) \in \Re e, \quad a^3 = \cos(\theta/2) \cos(\phi/2) \in \Re e, \quad (10)$$

and

$$a_{++} = \frac{\sqrt{(E+m)(E+p)}}{2\sqrt{2}m}, \quad a_{+-} = \frac{\sqrt{(E+m)(E-p)}}{2\sqrt{2}m}, \quad (11)$$

$$a_{-+} = \frac{\sqrt{(E-m)(E+p)}}{2\sqrt{2}m}, \quad a_{--} = \frac{\sqrt{(E-m)(E-p)}}{2\sqrt{2}m}. \quad (12)$$

However,  $A^\dagger A + B^\dagger B = 1$ , so the transformation matrix  $\mathcal{U}$  is unitary.

We now investigate the properties of the helicity-basis 4-spinors with respect to the discrete symmetry operations  $P$  and  $C$ . It is expected that  $\lambda \rightarrow -\lambda$  under parity, as in [1]. With respect to  $\mathbf{p} \rightarrow -\mathbf{p}$  the helicity 2-eigenspinors transform as follows:  $\phi_{\uparrow\downarrow} \Rightarrow -i\phi_{\downarrow\uparrow}$ , ref. [19]. Hence,

$$Pu_{\uparrow}(-\mathbf{p}) = -iu_{\downarrow}(\mathbf{p}), \quad Pv_{\uparrow}(-\mathbf{p}) = +iv_{\downarrow}(\mathbf{p}), \quad (13)$$

$$Pu_{\downarrow}(-\mathbf{p}) = -iu_{\uparrow}(\mathbf{p}), \quad Pv_{\downarrow}(-\mathbf{p}) = +iv_{\uparrow}(\mathbf{p}). \quad (14)$$

Thus, on the level of classical fields, we observe that the helicity 4-spinors transform to the 4-spinors of the opposite helicity. The charge conjugation operation is defined as  $C = \begin{pmatrix} 0 & \Theta \\ -\Theta & 0 \end{pmatrix} \mathcal{K}$ . Hence, we observe

$$Cu_{\uparrow}(\mathbf{p}) = -v_{\downarrow}(\mathbf{p}), \quad Cv_{\uparrow}(\mathbf{p}) = +u_{\downarrow}(\mathbf{p}), \quad (15)$$

$$Cu_{\downarrow}(\mathbf{p}) = +v_{\uparrow}(\mathbf{p}), \quad Cv_{\downarrow}(\mathbf{p}) = -u_{\uparrow}(\mathbf{p}). \quad (16)$$

due to the properties of the Wigner operator  $\Theta\phi_{\uparrow}^* = -\phi_{\downarrow}$  and  $\Theta\phi_{\downarrow}^* = +\phi_{\uparrow}$ . This is similar to the textbook case. For the  $CP$  (and  $PC$ ) operation we get:

$$CPu_{\uparrow}(-\mathbf{p}) = -PCu_{\uparrow}(-\mathbf{p}) = +iv_{\uparrow}(\mathbf{p}), \quad CPu_{\downarrow}(-\mathbf{p}) = -PCu_{\downarrow}(-\mathbf{p}) = -iv_{\downarrow}(\mathbf{p}), \quad (17)$$

$$CPv_{\uparrow}(-\mathbf{p}) = -PCv_{\uparrow}(-\mathbf{p}) = +iu_{\uparrow}(\mathbf{p}), \quad CPv_{\downarrow}(-\mathbf{p}) = -PCv_{\downarrow}(-\mathbf{p}) = -iu_{\downarrow}(\mathbf{p}), \quad (18)$$

which are different from the Dirac ‘common-used’ case. Similar conclusions can be drawn in the Fock space.

### 3. The $(1, 0) \oplus (0, 1)$ case.

In this Section we are going to investigate the behaviours of the field functions of the  $(1, 0) \oplus (0, 1)$  representation in the helicity basis with respect to  $P$ ,  $C$  and  $CP$  operations.

Let us start from the Klein-Gordon equation written for the 3-component function ( $\hbar = c = 1$ ):

$$(E^2 - \mathbf{p}^2)\psi_{(3)} = m^2\psi_{(3)}. \quad (19)$$

The equation (19) can be re-written in the form:

$$(E - \mathbf{S} \cdot \mathbf{p})(E + \mathbf{S} \cdot \mathbf{p})_{ij}\psi^j - p_i p_j \psi^j = m^2 \psi^i. \quad (20)$$

In the coordinate space it is of the second order in the time derivative, but as in the spin-1/2 case [21] we can reduce it to the set of the 3-“spinor” equations of the first orders. We can denote:

$$(E + \mathbf{S} \cdot \mathbf{p})\psi = m\xi, \quad p^i p^j \psi^j = \mathbf{p}(\mathbf{p} \cdot \psi) = m\mathbf{p} \varphi. \quad (21)$$

Hence, the equation (20) is written as

$$m(E - \mathbf{S} \cdot \mathbf{p})\xi - m\mathbf{p} \varphi = m^2\psi. \quad (22)$$

Now, we define  $\psi = \mathbf{E} - i\mathbf{B}$ . We can obtain (cf. with ref. [9])

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = -m \cdot \text{Im}(\xi), \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = m \cdot \text{Re}(\xi), \quad (23)$$

$$\nabla \cdot \mathbf{B} = -m \cdot \text{Re}(\varphi) + \text{const}_x, \quad \nabla \cdot \mathbf{E} = -m \cdot \text{Im}(\varphi) + \text{const}_x, \quad (24)$$

respectively, by means of separation of the equations in (21) into the real and imaginary parts. Next, we fix  $\varphi = im\phi$  and  $\xi = im\mathbf{A}$ , with  $\phi$  and  $\mathbf{A}$  being the electromagnetic-like potentials. The well-known Proca equation follows  $\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0$ . For the sake of completeness let us substitute  $\varphi$  and  $\xi$  in the equation (22). The result is  $-\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi = \mathbf{E}$  y  $\nabla \times \mathbf{A} = \mathbf{B}$ , that is equivalent to the second Proca equation  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ . We also can take the complex conjugates of the equations (21,22) and now define  $\chi = \mathbf{E} + i\mathbf{B}$ . As a result we have

$$(E - \mathbf{S} \cdot \mathbf{p})\chi = -m\xi \quad \text{or} \quad (E - \mathbf{S} \cdot \mathbf{p})(\mathbf{E} + i\mathbf{B}) = -im^2\mathbf{A}, \quad (25)$$

$$p^i p^j \chi^j = \mathbf{p}(\mathbf{p} \cdot \chi) = -m\mathbf{p}\varphi \quad \text{or} \quad \mathbf{p}[\mathbf{p} \cdot (\mathbf{E} + i\mathbf{B})] = -im^2\mathbf{p}\phi, \quad (26)$$

$$(E + \mathbf{S} \cdot \mathbf{p})\xi - \mathbf{p}\varphi = -m\chi \quad \text{or} \quad (E + \mathbf{S} \cdot \mathbf{p})\mathbf{A} - \mathbf{p}\phi = i(\mathbf{E} + i\mathbf{B}), \quad (27)$$

It is possible to put the above equations in the Kemmer  $10 \times 10$  matrix form (cf. [23]). The equation contains the part corresponding to the 4-vector potential and to fields. Taking into account the Proca equations, the definitions of  $\mathbf{E}^i = F^{i0}$ ,  $\mathbf{B}^i = -\frac{1}{2}\epsilon^{ijk}F^{jk}$  and the definition of the Levi-Civita tensor, we can obtain the Tucker-Hammer equation [4] from the Duffin-Kemmer-Petiau set of equations:

$$\begin{pmatrix} E^2 - \mathbf{p}^2 - 2m^2 & E^2 - \mathbf{p}^2 + 2E(\mathbf{S} \cdot \mathbf{p}) + 2(\mathbf{S} \cdot \mathbf{p})^2 \\ E^2 - \mathbf{p}^2 - 2E(\mathbf{S} \cdot \mathbf{p}) + 2(\mathbf{S} \cdot \mathbf{p})^2 & E^2 - \mathbf{p}^2 - 2m^2 \end{pmatrix} \begin{pmatrix} \chi \\ \psi \end{pmatrix} = 0. \quad (28)$$

In the covariant form the equation (28) is written:

$$(\gamma^{\mu\nu} p_\mu p_\nu + p^\mu p_\mu - 2m^2) \Psi_{(6)}(p^\mu) = 0. \quad (29)$$

with the  $6 \times 6$  Barut-Muzinich-Williams matrices [22]):

$$\gamma^{00} = \begin{pmatrix} 0 & 1_{3 \times 3} \\ 1_{3 \times 3} & 0 \end{pmatrix}, \quad \gamma^{i0} = \gamma^{0i} = \begin{pmatrix} 0 & -S^i \\ S^i & 0 \end{pmatrix}, \quad \gamma^{ij} = \begin{pmatrix} 0 & -\delta_{ij} + S_i S_j + S_j S_i \\ -\delta_{ij} + S_i S_j + S_j S_i & 0 \end{pmatrix}. \quad (30)$$

In the coordinate space we have  $(\gamma^{\mu\nu} \partial_\mu \partial_\nu + \partial^\mu \partial_\mu + 2m^2) \Psi(x^\mu) = 0$ . If we set the condition  $\partial_\mu \partial_\mu \rightarrow -m^2$  we can recover the Weinberg equation, ref. [3]:

$$\Gamma \begin{pmatrix} \chi \\ \psi \end{pmatrix} = \begin{pmatrix} -m^2 & m^2 + 2E(\mathbf{S} \cdot \mathbf{p}) + 2(\mathbf{S} \cdot \mathbf{p})^2 \\ m^2 - 2E(\mathbf{S} \cdot \mathbf{p}) + 2(\mathbf{S} \cdot \mathbf{p})^2 & -m^2 \end{pmatrix} \begin{pmatrix} \chi \\ \psi \end{pmatrix} = 0, \quad (31)$$

which is in the covariant form  $(\gamma^{\mu\nu} \partial_\mu \partial_\nu + m^2) \Psi(x^\mu) = 0$ . Thus, from what we have seen above, we can conclude that the Duffin-Kemmer-Petiau, Proca, Weinberg and Tucker-Hammer equations are all related one another. Let us consider the equation (28) as a set of equations for the bivector components in the helicity basis. Then, we have ( $p = |\mathbf{p}|$ ):

$$u_{1,\uparrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{E+p}{m} \chi_\uparrow \\ \frac{m}{E+p} \chi_\uparrow \end{pmatrix}, \quad u_{1,\rightarrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_\rightarrow \\ \chi_\rightarrow \end{pmatrix}, \quad u_{1,\downarrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{m}{E+p} \chi_\downarrow \\ \frac{E+p}{m} \chi_\downarrow \end{pmatrix}, \quad (32)$$

$$v_{1,\uparrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{E+p}{m} \chi_{\uparrow} \\ -\frac{m}{E+p} \chi_{\uparrow} \end{pmatrix}, v_{1,\rightarrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_{\rightarrow} \\ -\chi_{\rightarrow} \end{pmatrix}, v_{1,\downarrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{m}{E+p} \chi_{\downarrow} \\ -\frac{E+p}{m} \chi_{\downarrow} \end{pmatrix}, \quad (33)$$

where the 3-“spinors” are in the helicity basis (see [18, p.192]):

$$\chi_{\uparrow} = \begin{pmatrix} \frac{1+\cos\theta}{2} e^{-i\phi} \\ \frac{\sin\theta}{\sqrt{2}} \\ \frac{1-\cos\theta}{2} e^{i\phi} \end{pmatrix}, \chi_{\rightarrow} = \begin{pmatrix} -\frac{\sin\theta}{\sqrt{2}} e^{-i\phi} \\ \cos\theta \\ \frac{\sin\theta}{\sqrt{2}} e^{i\phi} \end{pmatrix}, \chi_{\downarrow} = \begin{pmatrix} \frac{1-\cos\theta}{2} e^{-i\phi} \\ -\frac{\sin\theta}{\sqrt{2}} \\ \frac{1+\cos\theta}{2} e^{i\phi} \end{pmatrix}. \quad (34)$$

The normalization condition is chosen  $\chi^{\dagger}\chi = 1$ .

Now we are ready to study the discrete symmetry operations for the spin-1 case (as we did for the spin-1/2 case in the previous Section). The bivectors have the following properties:

1. The Parity ( $\mathbf{p} \rightarrow -\mathbf{p}$ ,  $\theta \rightarrow \pi - \theta$ ,  $\phi \rightarrow \pi + \phi$ ). We note that the 3-“spinors” are transformed as  $\chi_h \rightarrow -\chi_{-h}$ ; the parity operator is  $P = \gamma^{00}$  (it is analogous to that which was used for spin-1/2). Therefore,

$$Pu_{1,\uparrow}(-\mathbf{p}) = -u_{1,\downarrow}(\mathbf{p}), Pu_{1,\rightarrow}(-\mathbf{p}) = -u_{1,\rightarrow}(\mathbf{p}), Pu_{1,\downarrow}(-\mathbf{p}) = -u_{1,\uparrow}(\mathbf{p}) \quad (35)$$

$$Pv_{1,\uparrow}(-\mathbf{p}) = +v_{1,\downarrow}(\mathbf{p}), Pv_{1,\rightarrow}(-\mathbf{p}) = +v_{1,\rightarrow}(\mathbf{p}), Pv_{1,\downarrow}(-\mathbf{p}) = +v_{1,\uparrow}(\mathbf{p}). \quad (36)$$

2. The Charge Conjugation is defined  $C = e^{i\alpha} \begin{pmatrix} 0 & \Theta \\ -\Theta & 0 \end{pmatrix} \mathcal{K}$ , with  $\Theta_{[1]} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ . Hence,  $\Theta\chi_{\uparrow}^* = \chi_{\downarrow}$ ,  $\Theta\chi_{\downarrow}^* = \chi_{\uparrow}$ ,  $\Theta\chi_{\rightarrow}^* = -\chi_{\rightarrow}$ . Finally, we have

$$Cu_{1,\uparrow}(\mathbf{p}) = +e^{i\alpha}v_{1,\downarrow}(\mathbf{p}), Cu_{1,\rightarrow}(\mathbf{p}) = -e^{i\alpha}v_{1,\rightarrow}(\mathbf{p}), Cu_{1,\downarrow}(\mathbf{p}) = +e^{i\alpha}v_{1,\uparrow}(\mathbf{p}) \quad (37)$$

$$Cv_{1,\uparrow}(\mathbf{p}) = -e^{i\alpha}u_{1,\downarrow}(\mathbf{p}), Cv_{1,\rightarrow}(\mathbf{p}) = +e^{i\alpha}u_{1,\rightarrow}(\mathbf{p}), Cv_{1,\downarrow}(\mathbf{p}) = -e^{i\alpha}u_{1,\uparrow}(\mathbf{p}) \quad (38)$$

3. The  $CP$  and  $PC$  operations:

$$CPu_{1,\uparrow}(-\mathbf{p}) = -e^{i\alpha}v_{1,\uparrow}(\mathbf{p}), CPv_{1,\uparrow}(-\mathbf{p}) = -e^{i\alpha}u_{1,\uparrow}(\mathbf{p}), \quad (39)$$

$$CPu_{1,\downarrow}(-\mathbf{p}) = -e^{i\alpha}v_{1,\downarrow}(\mathbf{p}), CPv_{1,\downarrow}(-\mathbf{p}) = -e^{i\alpha}u_{1,\downarrow}(\mathbf{p}), \quad (40)$$

$$CPu_{1,\rightarrow}(-\mathbf{p}) = +e^{i\alpha}v_{1,\rightarrow}(\mathbf{p}), CPv_{1,\rightarrow}(-\mathbf{p}) = +e^{i\alpha}u_{1,\rightarrow}(\mathbf{p}). \quad (41)$$

We found within the classical field theory that the properties of a particle and an anti-particle of spin-1 are different comparing with the known cases (when the basis is chosen in such a way that the solutions are the eigenstates of the parity).

#### 4. The Conclusions.

Similarly to the  $(\frac{1}{2}, \frac{1}{2})$  representation [16], the  $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$  and  $(1, 0) \oplus (0, 1)$  field functions in the helicity basis are *not* the eigenstates of the common-used parity operator;  $|\mathbf{p}, \lambda\rangle \Rightarrow |-\mathbf{p}, -\lambda\rangle$  on the classical level. This is in accordance with the earlier consideration of Berestetskii, Lifshitz and Pitaevskii. Helicity field functions may satisfy the ordinary Dirac equation with  $\gamma$ 's to be in the spinorial representation. Helicity field

functions can be expanded in the set of the Dirac 4-spinors by means of the matrix  $\mathcal{U}^{-1}$  given in this paper.  $P$  and  $C$  operations anticommute in this framework on the classical level. The different formulations of the spin-1 particles are all connected by algebraic transformations. The properties of spin-1 solutions in the helicity basis with respect to  $P$ ,  $C$ ,  $CP$  are similar to those in the spin-1/2 case, and differ from the usual case.

In order to make the above conclusions to be more rigorous one should repeat the calculations in the Fock space within the “secondary quantization” framework (see [21] for the spin-1/2 case).

## References

- [1] V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, 1982), §16.
- [2] Yu. V. Novozhilov, *Introduction to Elementary Particle Physics* (Pergamon Press, 1975), §4.3, 6.2.
- [3] S. Weinberg, Phys. Rev. **133B** (1964) 1318.
- [4] R. H. Tucker y C. L. Hammer, Phys. Rev. D **3** (1971) 2448.
- [5] V. V. Dvoeglazov, Helv. Phys. Acta, **70** (1997) 677.
- [6] G. Ziino, Ann. Fond. Broglie **14** (1989) 427; ibid **16** (1991) 343; A. Barut and G. Ziino, Mod. Phys. Lett. **A8** (1993) 1011; G. Ziino, Int. J. Mod. Phys. **A11** (1996) 2081.
- [7] N. D. S. Gupta, Nucl. Phys. **B4** (1967) 147; D. V. Ahluwalia, Int. J. Mod. Phys. **A11** (1996) 1855; V. V. Dvoeglazov, Hadronic J. **20** (1997) 435.
- [8] V. V. Dvoeglazov, Mod. Phys. Lett. **A12** (1997) 2741.
- [9] V. V. Dvoeglazov, Spacetime and Substance **3**(12) (2002) 28; Rev. Mex. Fis., Supl. 1, **49** (2003) 99.
- [10] V. V. Dvoeglazov, Physica Scripta **64** (2001) 201.
- [11] V. V. Dvoeglazov, Hadronic J. **25** (2002) 137.
- [12] V. V. Dvoeglazov, Hadronic J. **26** (2003) 299.
- [13] A. Sankaranarayanan and R. H. Good, jr., Nuovo Cim. **36** (1965) 1303.
- [14] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Phys. Lett. **B316** (1993) 102; V. V. Dvoeglazov, Int. J. Theor. Phys. **37**, (1998) 1915, and references therein.
- [15] L. H. Ryder, *Quantum Field Theory*. (Cambridge University Press, 1985).
- [16] H. M. Ruck y W. Greiner, J. Phys. G: Nucl. Phys. **3** (1977) 657.
- [17] M. Jakob and G. C. Wick, Ann. Phys. **7** (1959) 404.
- [18] D. A. Varshalovich, A. N. Moskalev and V. K. Khersonskii, *Quantum Theory of Angular Momentum* (World Scientific, 1988), §6.2.5.
- [19] V. V. Dvoeglazov, Fizika **B6** (1997) 111.
- [20] J. J. Sakurai, *Advanced Quantum Mechanics*. (Addison-Wesley, 1967), §3.2; A. Gersten, Found. Phys. Lett. **12** (1999) 291; ibid. **13** (2000) 185; V. V. Dvoeglazov, J. Phys. A: Math. Gen. **33** (2000) 5011.
- [21] V. V. Dvoeglazov, In *Memorias de la 8a Reunión Nacional Académica de Física y Matemáticas, 12-16 de Mayo de 2003, ESFM-IPN, México, D.F.*, p. 45-54.
- [22] A. O. Barut, I. Muzinich and D. Williams, Phys. Rev. **130** (1963) 442.
- [23] W. Greiner, *Relativistic Quantum Mechanics*. The 1st English Ed. (Springer, 1990).